Vortex in a Bose-Einstein condensate with dipole-dipole interactions

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Introduction

Dipole-dipole interactions are both long-range and anisotropic and therefore of a quite different nature to the usual short-range s-wave interactions found in alkali BECs. A dipolar BEC (chromium) was made for the first time in Stuttgart in Nov 2004 [A. Griesmaier, J. Werner, S. Hensler, J. Stuhler and T. Pfau, PRL **94**, 160401 (2005).]. Here we study the effects of dipole-dipole interactions upon the rotational properties of a BEC. It will turn out that these effects depend on the shape (and boundary) of the BEC.

Do attractive bosons condense? -the role of interactions

for $U(r-r') = g\delta(r-r')$

$$E_{int} = \frac{g}{2}\sum_{ijkl}\langle \hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_k \hat{a}_l\rangle \int X_i^*(r)X_j^*(r)X_k(r)X_l(r)d^3r$$

Fragmented state $|F\rangle = \frac{(\hat{a}_1^\dagger)^{N_1}}{\sqrt{N_1!}}\frac{(\hat{a}_2^\dagger)^{N_2}}{\sqrt{N_2!}}|\text{vac}\rangle$ Coherent state $|\Phi\rangle = \frac{1}{\sqrt{N!2^N}}\left(\hat{a}_1^\dagger + e^{-i\phi}\hat{a}_2^\dagger\right)^N|\text{vac}\rangle$

For simplicity consider the case of just two bosons: A) in different states, and B) in the same state:

A) Symmetrized wave function for two bosons in different states:

$$\Psi(r,r') = \frac{1}{\sqrt{2}}[X_1(r)X_2(r') + X_1(r')X_2(r)]$$

$$E_{int} = 2g\int |X_1(r)|^2|X_2(r)|^2 d^3r$$

B) Wavefunction for two bosons in the same state:

$$\Psi(r,r') = X_1(r)X_1(r')$$

$$E_{int} = g\int |X_1(r)|^4 d^3r$$

Roughly speaking, the fragmented state interacts twice as strongly. When g>0 (repulsive) simple BEC is favoured, and when g<0 (attractive) fragmentation is favoured [Huang 1987, Nozieres, 1995].

Exact solution to hydrodynamic eqns for a dipolar: analogy with electrostatics

$$\Phi_{dd}(r) \equiv \int d^3r' U_{dd}(r-r')n(r')$$

but: $\frac{1-3\cos^2\theta}{4\pi r^3} = -\partial_z^2\frac{1}{4\pi r} - \frac{1}{3}\delta(r)$

so: $\Phi_{dd}(r) = -C_{dd}\left(\partial_z^2\phi(r) + \frac{1}{3}n(r)\right)$ where $\phi(r) = \frac{1}{4\pi}\int\frac{d^3r'n(r')}{|r-r'|}$ (A)

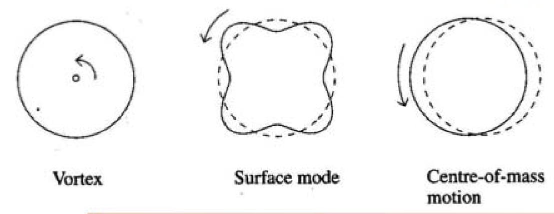
$\phi(r)$ obeys Poisson's eqn $\nabla^2\phi(r) = -n(r)$

So if $n(r)$ is parabolic then:

$$\phi(r) = a_0 + a_1x^2 + a_2y^2 + a_3z^2 + a_4x^2y^2 + a_5x^2z^2 + a_6y^2z^2 + a_7x^4 + a_8y^4 + a_9z^4$$

by (A) a parabolic density profile therefore also gives a parabolic dipole-dipole mean-field potential!!

Ways of adding angular momentum to a trapped BEC



$\nabla \times v_s = 0$

Attractive interactions favour centre of mass motion

Centre of mass motion at trap frequency: [W. Kohn, Phys. Rev. **123**, 1242 (1961)]

$$\psi(r,t) = \exp\left[-\frac{i\mu t}{\hbar}\right]\exp\left[i\left\{x - \frac{A(t)}{2}\right\}\frac{\beta(t)}{\hbar} + \left\{y - \frac{B(t)}{2}\right\}\frac{\gamma(t)}{\hbar}\right]\psi_0(x - A(t), y - B(t), z)$$

$$v_x = \frac{\beta}{m} = \dot{A}\ ,\quad \ddot{A} + \omega_x^2 A = 0;\qquad v_y = \frac{\gamma}{m} = \dot{B}\ ,\quad \ddot{B} + \omega_y^2 B = 0$$

Dipole-Dipole Interactions

Magnetic dipole-dipole interaction: magnetic moments μ of atoms aligned by external magnetic field

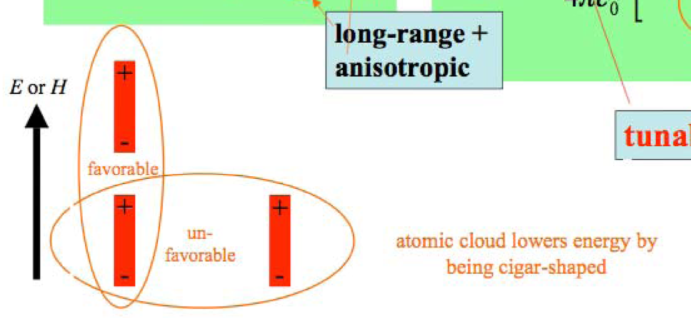
$$U_{dd}(r) = \frac{\mu_0\mu^2}{4\pi}\left[\frac{1-3\cos^2\theta}{r^3}\right]$$

Electric dipole-dipole interaction:
(i) permanent electric moments (polar molecules);
(ii) electric moments induced by external electric field E

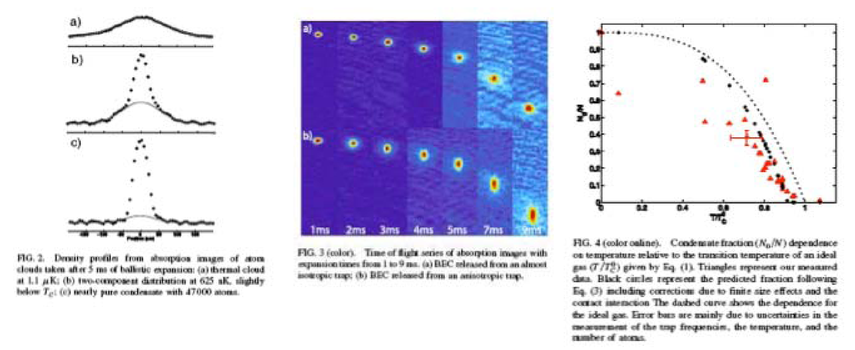
$$U_{dd}(r) = \frac{E^2\alpha^2}{4\pi\varepsilon_0}\left[\frac{1-3\cos^2\theta}{r^3}\right]$$

long-range + anisotropic

tunability

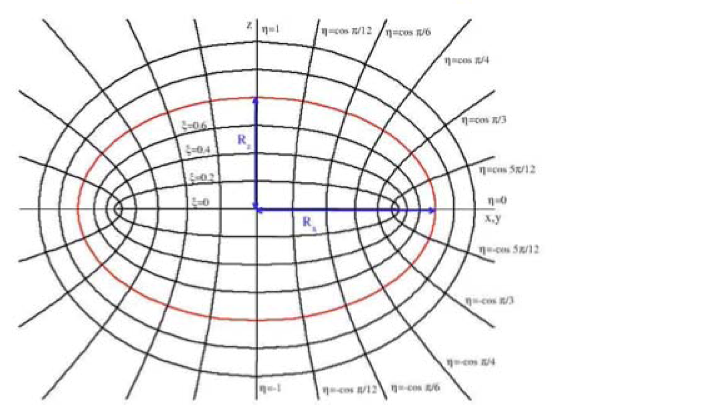


Chromium BEC in Stuttgart: Nov 2004

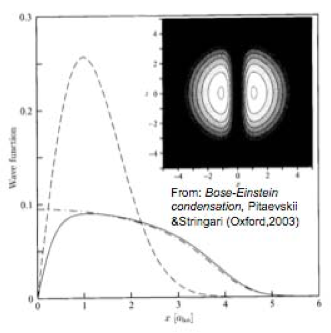


A. Griesmaier, J. Werner, S. Hensler, J. Stuhler and T. Pfau, PRL **94**, 160401 (2005).

Integration can be achieved either by using the known Green's function in spheroidal coords or by integrating over ellipsoids [G. Green, ***On the determination of Exterior and Interior attractions of ellipsoids of variable densities***, Trans. Camb. Phil. Soc. Vol V, Part III, 1835]



Vortex in a dipolar BEC



Usual strategy (e.g. Pitaevskii &Stringari (Oxford, 2003): solve Gross-Pitaevskii eqn for an infinite cylinder to obtain vortex-core size. Then adapt to trapped system. Result:

Vortex energy $E_v = \frac{4\pi}{3}n_0 R_z \frac{\hbar^2}{m}\ln\frac{0.67R_x}{\xi}$

Critical rotation frequency $\Omega_{cr} = \frac{5}{2}\frac{\hbar}{mR_x^2}\ln\frac{0.67R_x}{\xi}$

Problem for dipolar interactions: dipole-dipole interaction vanishes for uniform system:

$\Phi_{dd}(r) = -C_{dd}\left[\partial_z^2\phi(r) + \frac{1}{3}n(r)\right]$ where $\phi(r) = \frac{1}{4\pi}\int\frac{d^3r'n(r')}{|r-r'|}$

Our strategy: minimize total energy functional based on ansatz

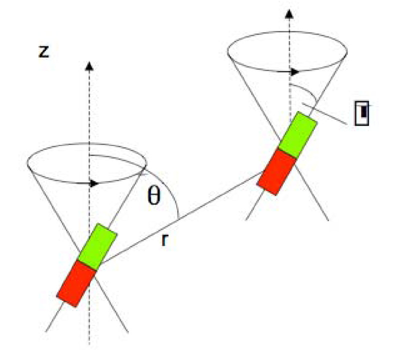
$$n(r) = n_0\frac{\rho^2}{\rho^2+\beta^2}\left(1 - \frac{\rho^2}{R_x^2} - \frac{z^2}{R_z^2}\right)$$

Where β is the size of the vortex core.

Controlling dipole-dipole interactions by rapidly rotating the external field

[Giovanazzi, Gorlitz & Pfau PRL 89, 130401 (2002)]

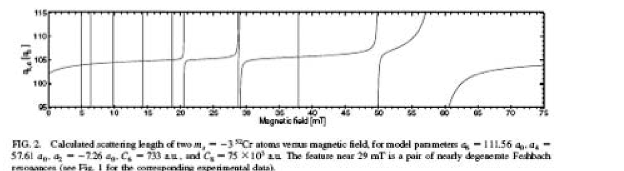
The sign of the interaction can be reversed, or the interaction can even be averaged out completely when $\varphi = 54.7°$ (the 'magic angle')



$$B(t) = B_0[\cos(\varphi)\hat{z} + \sin(\varphi)\{\cos(\Omega t)\hat{x} + \sin(\Omega t)\hat{y}\}] \Rightarrow \langle U_{dd}\rangle_t = \frac{\mu_0\mu^2}{4\pi}\left(\frac{3\cos^2\varphi - 1}{2}\right)\frac{1-3\cos^2\vartheta}{r^3}$$

where $\omega_{Larmor} >> \Omega >> \omega_{trap}$

Controlling the short-range interactions: Feshbach scattering resonances in chromium



J. Werner, A. Griesmaier, S. Hensler, J. Stuhler and T. Pfau, PRL **94**, 183201 (2005).

The exact solution to dipolar hydrodynamic equations

[D O'D, S. Giovanazzi, C Eberlein, PRL **92**, 250401 (2004)]

$$n(r,t) = n_0(t)\left[1 - \frac{x^2}{R_x^2(t)} - \frac{y^2}{R_y^2(t)} - \frac{z^2}{R_z^2(t)}\right]$$

'Electrostatic' integration gives dipolar mean-field potential:

$$\Phi_{dd}(\rho,z) = \frac{n_0 C_{dd}}{3}\left[\frac{\rho^2}{R_x^2} - \frac{2z^2}{R_z^2} - f(\kappa)\left(1 - \frac{3}{2}\frac{\rho^2 - 2z^2}{R_x^2 - R_z^2}\right)\right]$$

where $\kappa = R_x/R_z$, and $f(\kappa) = \frac{1+2\kappa^2}{1-\kappa^2} - \frac{3\kappa^2 \text{arctanh}\sqrt{1-\kappa^2}}{(1-\kappa^2)^{3/2}}$

Hydrodynamic equations give BEC radii:

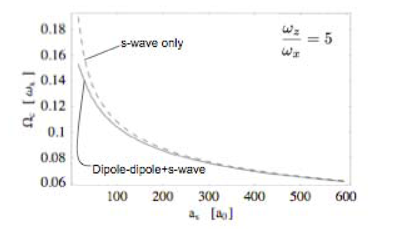
$$R_x = R_y = \left[\frac{15gN\kappa}{4\pi m\omega_x^2}\left\{1 + \varepsilon_{dd}\left(\frac{3}{2}\frac{\kappa^2 f(\kappa)}{1-\kappa^2} - 1\right)\right\}\right]^{1/5},\ \text{and}\ R_z = R_x/\kappa$$

Aspect ratio κ of BEC given by solution of a transcendental equation:

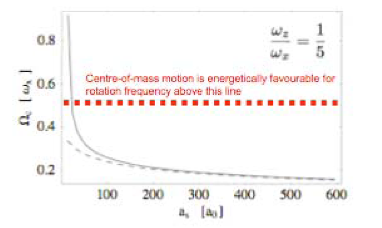
$$\frac{\kappa^2}{\gamma^2}\left[\frac{3\varepsilon_{dd}f(\kappa)}{1-\kappa^2}\left(\frac{\gamma^2}{2}+1\right) - 2\varepsilon_{dd} - 1\right] = \varepsilon_{dd} - 1\ ,\quad \text{where}\ \gamma = \omega_z/\omega_x$$

Vortex in a dipolar BEC: results

Oblate (pancake) BEC: dipole-dipole interactions are predominantly repulsive



Prolate (cigar) BEC: dipole-dipole interactions are predominantly attractive



Energy for centre-of-mass motion having angular momentum L=Nh/2π is $E_{com}^{L=1} = \frac{1}{2}Nm\omega_x^2 a_{ho}^2 = \frac{1}{2}N\hbar\omega_x$

Attractive part of dipole-dipole interactions causes the centre-of-mass motion to be favoured instead of a vortex when the BEC becomes prolate enough and the dipole-dipole interactions are strong enough

Relative magnitude of dipole-dipole and s-wave interactions

long-range: $U_{dd}(r) = \frac{C_{dd}}{4\pi}\left[\frac{1-3\cos^2\theta}{r^3}\right]$

short-range: $U_s(r) = \frac{4\pi\hbar^2 a}{m}\delta(r) \equiv g\delta(r)$

Dimensionless parameter characterizing relative magnitude of dd and s-wave interactions $\varepsilon_{dd} \equiv \frac{C_{dd}}{3g}$ $\varepsilon_{dd} > 1 \Rightarrow$ collapse

Magnetic dipole-dipole:

^{87}Rb	$\varepsilon_{dd} = 0.007$	^{52}Cr	$\varepsilon_{dd} = 0.144$
Na	$\varepsilon_{dd} = 0.004$	^{50}Cr	$\varepsilon_{dd} = 0.360$

MOLECULES?....dipole moment ~1 Debye.... $\varepsilon_{dd} \approx 100$ is possible.

Collisionless hydrodynamics of a superfluid at T=0

$\psi(r,t) = \sqrt{n(r,t)}\exp i\phi(r,t)$, Potential flow: $v(r,t) = \frac{\hbar}{m}\nabla\phi(r,t)$

Continuity eqn $\frac{\partial n}{\partial t} = -\nabla.(nv)$, $m\frac{\partial v}{\partial t} = -\nabla\left[\frac{mv^2}{2} + V_{trap} + gn + \Phi_{dd}\right]$ Euler eqn

where $\Phi_{dd}(r) = \int U_{dd}(r-r')n(r')\,d^3r'$ Mean-field potential due to dipole-dipole interactions

Static solution when $\Phi_{dd} = 0$: inverted parabola density profile

$$n(r) = |\psi(r)|^2 = \frac{\mu - V_{trap}(r)}{g}\qquad V_{trap}(r) = \frac{1}{2}m[\omega_x^2x^2 + \omega_y^2y^2 + \omega_z^2z^2]$$

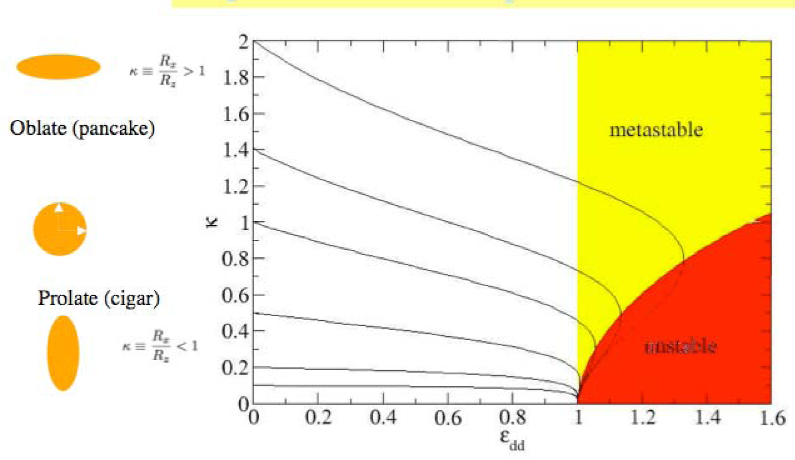
Dynamic solution when $\Phi_{dd} = 0$: scaling parabola

$$n(r,t) = n_0(t)\left[1 - \frac{x^2}{R_x^2(t)} - \frac{y^2}{R_y^2(t)} - \frac{z^2}{R_z^2(t)}\right],\quad n_0(t) = 15N/[8\pi R_x(t)R_y(t)R_z(t)]$$

$$v(r,t) = (1/2)\nabla[\alpha_x(t)x^2 + \alpha_y(t)y^2 + \alpha_z(t)z^2],\ \text{where}\ \alpha_j = \dot{R}_j/R_j$$

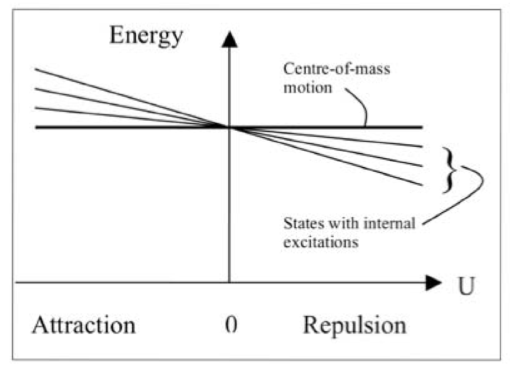
solution when $\Phi_{dd} \neq 0$ ..... $n(r) = ??$

Aspect ratio of a dipolar BEC



CONCLUSION

Quantum phase transition as interactions become net positive, i.e. qualitatively different behavior under rotation depending upon whether the interactions are positive or negative



Future work:
fragmentation in dipolar BECs