Semiclassical Coupled Wave Theory for TM-waves in 1-D Photonic Crystals

G. V. Morozov and D. W. L. Sprung
Department of Physics and Astronomy, McMaster University
Hamilton, Ontario L8S 4M1 Canada

J. Martorell
Departament d'Estructura i Constituents de la Materia,
Facultat Física, University of Barcelona
Barcelona 08028, Spain
(Dated: January 14, 2003)

Semiclassical coupled wave theory is extended to TM waves in one-dimensional periodic dielectric structures. Using this theory, the band widths and reflection/transmission characteristics of such structures, as functions of the incident wave frequency, are in good agreement with exact numerical simulations even for very deep gratings.

PACS numbers: 03.50.De, 03.65.Sq, 42.70.Qs

I. INTRODUCTION

A one dimensional periodic structure with high refractive index contrast can serve as a photonic crystal [1–5]. Among the common theoretical methods for wave motion in such structures, the coupled wave approach offers superior physical insight and gives simple analytical results in limiting cases. Unfortunately, the conventional coupled wave theory of Kogelnik [6–8] fails in the case of high refractive index contrast, which is required for a 1D photonic crystal.

In a previous article [9] we extended the first approximation of the semiclassical coupled wave theory for normal propagation [10, 11] to the case of oblique incidence of TE waves. Using the Bogolyubov averaging method [12], we developed a second approximation for the same case. Our method allows for both variable amplitudes and variable (geometric-optics) phases in the counter-propagating waves. While it is analytically almost as simple as conventional coupled wave theory, our method is essentially exact for any achievable ratio (e.g. 1.5 : 4.6) of the indices of refraction of the layers comprising buildable devices.

In this paper we extend our semiclassical coupled wave theory to the case of oblique propagation of TM electromagnetic waves. Both the first and the second approximations are extracted. Surprisingly, the first approximation works even better for the TM case than it did for TE waves. The second approximation gives practically exact results even for structures with a very deep modulation of the refractive index. Using our analytic expressions for the band edges, one can easily optimize the positions and widths of the forbidden zones, in order to fine tune photonic devices.

In the following section, the semiclassical coupled-wave method for TM waves is developed. We obtain simple analytical expressions for the Bloch phase, which is a key parameter for determination of band structure, and for the reflection/transmission amplitudes. As an application, in Sec. III we find the optimal omnidirectional forbidden band of a bilayer periodic dielectric structure. In Sec. IV we show how to apply the theory to a periodic structure with a continuous harmonic profile of the refractive index. The conclusions are in Sec. V.

II. SEMICLASSICAL COUPLED WAVE THEORY

We consider a wide, absorptionless, non-magnetic ($\mu = 1$) slab whose normal is the z-axis, occupying the region $0 < z < L$. The index of refraction $n(z) = n(z+d)$ varies periodically in the z-direction, but does not depend on x or y. The slab is surrounded on both sides by a homogeneous dielectric medium with $n(z) = n_0$ on the left, and $n(z) = n_f$ on the right. In gaussian units, the dielectric permittivity $\epsilon(z)$ is the square of the refractive index: $\epsilon(z) = n^2(z)$. For monochromatic fields of circular frequency ω, i.e. for harmonic time dependence, we can set $\mathbf{E}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}) \exp(-i\omega t)$ and $\mathbf{H}(\mathbf{r}, t) = \mathbf{H}(\mathbf{r}) \exp(-i\omega t)$.

TM waves have $\mathbf{H}$ perpendicular to the plane of wave propagation, which can be chosen as the xz plane, when the refractive index varies only in the z-direction. Writing $\mathbf{H}(\mathbf{r}) = H(z)\, e^{i\,k\beta x}\hat{\mathbf{e}}_y$, Maxwell's equations inside the periodic slab reduce to a single scalar equation for the amplitude $H(z) = H_y$

$$\frac{d}{dz}\left(\frac{1}{\epsilon(z)}\frac{dH}{dz}\right) + k^2\left[1 - \frac{\beta^2}{\epsilon(z)}\right]H(z) = 0 \qquad (1)$$

or, in terms of the refractive index

$$\frac{d^2H}{dz^2} + k^2[n^2(z) - \beta^2]H(z) - \frac{2}{n(z)}\frac{dn(z)}{dz}\frac{dH}{dz} = 0\,. \qquad (2)$$

In these formulae, $k = \omega/c$, and $k\beta$ is the (constant) x-component of the wave vector of modulus $k(z) = \omega\, n(z)/c$ inside the medium. If a TM wave impinges on the periodic medium from the region $z < 0$, then $\beta = n_0 \sin\theta_0$, where θ_0 is the angle of incidence measured from the normal. According to Maxwell's equations, the components $E_x(z)$ and $E_z(z)$ of the electric field of the TM wave $\mathbf{E}(\mathbf{r}) = [E_x(z)\hat{\mathbf{e}}_x + E_z(z)\hat{\mathbf{e}}_z]\, e^{i\,k\beta x}$ can be expressed as

$$E_x(z) = -\frac{i}{k\epsilon(z)}\frac{dH(z)}{dz}, \quad E_z(z) = -\frac{\beta}{\epsilon(z)}H(z)\,. \qquad (3)$$

We require the fields $H_y(z)$ and $E_x(z)$ to be continuous. Therefore, at any point of discontinuity z_j of the dielectric permittivity $\epsilon(z)$, or correspondingly of the refractive index $n(z)$, we match $H(z)$ and $n^{-2}(z)dH(z)/dz$. By means of the substitution $H(z) = h(z)n(z)$, we eliminate the first derivative term from eq. (2):

$$\frac{d^2h(z)}{dz^2} + \left[k^2(n^2(z) - \beta^2) + Q(z)\right] h(z) = 0, \qquad (4)$$

where

$$Q(z) = \frac{1}{n(z)}\frac{d^2n(z)}{dz^2} - \frac{2}{n^2(z)}\left(\frac{dn}{dz}\right)^2. \qquad (5)$$

Eq.(4), (as well as eq. (2) of Ref. [9] for TE polarization,) is formally equivalent to the wave equation for normal incidence if we were to follow Ginzburg [13] in setting the effective value of $n(z)$ to

$$\begin{aligned} n_e^s(z) &= \sqrt{n^2(z) - \beta^2}\,, \\ n_e^p(z) &= \sqrt{n^2(z) - \beta^2 + \frac{Q(z)}{k^2}} \end{aligned} \qquad (6)$$

for TE and TM polarizations respectively. However, $n_e^p(z)$ would then depend on the vacuum wave number k and it would be quite difficult to extend our semiclassical coupled wave theory to the TM case. Instead, let us use the same effective value of $n(z)$ for TM waves as for TE waves. Introducing the notation $n_e = n_e^s(z)$, we seek a solution of eq. (4) in terms of two counterpropagating waves with slowly varying amplitudes $A^{\pm}(z)$ and geometric-optics phases

$$\begin{aligned} h(z) &= \frac{A^{(+)}(z)e^{i\psi(z)}}{\sqrt{n_e(z)}} + \frac{A^{(-)}(z)e^{-i\psi(z)}}{\sqrt{n_e(z)}}\,, \\ \psi(z) &= k\int_0^z n_e(z')dz'\,. \end{aligned} \qquad (7)$$

After the substitution of expression (7), eq. (4) (and, therefore, Maxwell's equations) becomes an identity if the amplitudes $A^{(\pm)}(z)$ satisfy the system

$$\begin{aligned} \frac{dA^{(+)}(z)}{dz} &= S^{(-)}(z)A^{(-)}(z)\,, \\ \frac{dA^{(-)}(z)}{dz} &= S^{(+)}(z)A^{(+)}(z)\,, \end{aligned} \qquad (8)$$

where

$$S^{(\pm)}(z) = w(z)e^{\pm 2i\psi(z)}\,, \qquad (9)$$

and the function $w(z)$ satisfies the Riccati equation

$$\begin{aligned} \frac{dw}{dz} - \frac{1}{n_e}\frac{dn_e}{dz}\,w + w^2 &= \frac{1}{2}\frac{1}{n_e}\frac{d^2n_e}{dz^2} - \frac{3}{4}\frac{1}{n_e^2}\left(\frac{dn_e}{dz}\right)^2 \\ &+ \frac{2}{n^2}\left(\frac{dn}{dz}\right)^2 - \frac{1}{n}\frac{d^2n}{dz^2}\,. \end{aligned} \qquad (10)$$

This has an exact solution

$$w(z) = \frac{1}{2}\frac{1}{n_e}\frac{dn_e}{dz} - \frac{1}{n}\frac{dn}{dz}\,. \qquad (11)$$

For TE waves, (see Ref. [9]), the analogous procedure leads to the same form (8) of $S^{(\pm)}(z)$ but with $w(z) = 1/(2\,n_e(z))dn_e/dz$. The system (8) is exact. Introducing the phase averaged refractive index $n_{e,av} = \psi(d)/kd$, i.e.

$$n_{e,av} = \frac{1}{d}\int_0^d n_e(z')dz' \equiv \frac{1}{d}\int_0^d \sqrt{n^2(z') - \beta^2}\,dz'\,, \qquad (12)$$

we find that the quantities $S^{(\pm)}(z)\exp(\mp 2ikn_{e,av}z)$ are periodic functions that can be Fourier expanded as

$$S^{(\pm)}(z)e^{\mp 2ikn_{e,av}z} = \sum_{m=-\infty}^{m=+\infty} p_m^{(\pm)} e^{i2\pi mz/d}\,, \qquad (13)$$

where

$$\begin{aligned} p_m^{(\pm)} &= \frac{1}{d}\mathcal{P}\int_0^d \left(\frac{1}{2n_e(z)}\frac{dn_e}{dz} - \frac{1}{n(z)}\frac{dn}{dz}\right) e^{2i(\pm\psi(z)\mp kn_{e,av}z - \pi mz/d)}dz \\ &+ \frac{1}{2d}\sum_j \ln\left(\frac{n_e(z_j+0)n^2(z_j-0)}{n_e(z_j-0)n^2(z_j+0)}\right)e^{2i(\pm\psi(z_j)\mp kn_{e,av}z_j - \pi mz_j/d)}\,. \end{aligned} \qquad (14)$$

Physically, these coefficients represent the importance of coupling between the two counterpropagating waves of (7) due to the m-th Fourier components of the functions $S^{\pm}(z)$. The $\mathcal{P}$ implies a principal value integral, and the sum over $j=1,2,...$ takes into account the contribution to $p_m^{(\pm)}$ of jumps in the refractive index $n(z)$ at the points of discontinuity z_j within the period. If a discontinuity in $n(z)$ occurs at the beginning or at the end of a period, we should take this discontinuity into account only once, say at the beginning of the period. The quantities $n(z_j \pm 0)$ are the limiting values of the refractive index $n(z)$ to the right/left of a point of discontinuity z_j. We see that $p_{-m}^{(+)}$ is just the complex conjugate of $p_m^{(-)}$, so from this point forward we will use the notation $p_m^{(-)} \equiv p_m$ and $p_{-m}^{(+)} = p_m^*$. These coefficients depend on the wavenumber k of the incident wave, on the behaviour of the slab refractive index $n(z) = n(z+d)$ over a period d, and on the external conditions (n_0, θ_0). However, in case of normal incidence ($\theta_0 = 0$) the dependence on n_0 drops out. In conventional coupled wave theory the magnitudes of the coupling coefficients for TM case are determined by

$$\begin{aligned} p_m^{con} &= \frac{k}{2d}\,\frac{1-2\beta/\varepsilon_{av}}{\sqrt{\varepsilon_{av}-\beta^2}}\int_0^d \varepsilon(z)e^{-2i\pi mz/d}dz\,, \\ \varepsilon_{av} &= \frac{1}{d}\int_0^d \varepsilon(z)dz\,. \end{aligned} \tag{15}$$

The difference between the coupling coefficients p_m and p_m^{con} is the key point of departure of our semiclassical theory from the conventional (Kogelnik) one and is due to the fact that in our theory multiwave diffraction by periodic inhomogeneities of $\varepsilon(z)$ is taken into account. In Kogelnik theory only one diffracted wave: $\exp(-ikz\sqrt{\varepsilon_{av}-\beta^2})$ (in addition to the transmitted wave: $\exp(ikz\sqrt{\varepsilon_{av}-\beta^2})$) was assumed to exist within the periodic structure.

In order to find the band gaps and transmission/reflection coefficients of the structure we must solve the system (8). As in Ref. [9], we define the Bragg resonances k_q of our periodic slab, by

$$k_q n_{e,av} = \frac{\pi}{d}\,q\,, \quad q = 1,2,3... \tag{16}$$

and introduce the detuning δ_q from the q-th Bragg resonance, as

$$k n_{e,av} = \frac{\pi}{d}\,q + \delta_q,\ \begin{cases} -\frac{\pi}{2d} < \delta_{q\neq 1} < \frac{\pi}{2d} \\ -\frac{\pi}{d} < \delta_1 < \frac{\pi}{2d} \end{cases}. \tag{17}$$

Eq. (16) is the well-known Bragg condition for constructive interference. Physically, it means that the optical path difference between partial waves reflected from successive planes of the inhomogeneous refractive index contains an integral number of wavelengths. In the initial ("zeroth") approximation the Bragg resonances coincide with the centers of the forbidden bands. In conventional coupled wave theory the Bragg resonances are located at points where $k_q d\sqrt{\varepsilon_{av}-\beta^2} = \pi\, q$. This leads to a less accurate determination of the centers of the forbidden bands k_q and, as a result, to a less accurate estimation of the detuning δ_q. This is the second point of departure between the two theories.

If all the coefficents $|p_m d| < 1$ and the detuning $|\delta_q d| < 1$, we can use the method of averaging [12] to obtain an approximate solution of (8). In practice, the method assumes that the main contribution to the exact solutions of (8) is provided by the slowly-varying components of the functions $S^{\pm}(z)$. (We also note that the method of averaging can give reasonable results even in cases where some of $|p_m d| > 1$ or $|\delta_q d| > 1$.) Repeating the calculations of Ref. [9], but taking into account that the coefficients p_m for TM polarization are determined by (14) and, therefore, differ from the TE case, we find that in each zone along the k-axis $\pi(-\frac{1}{2}+q)/(n_{e,av}d) < k < \pi(\frac{1}{2}+q)/(n_{e,av}d)$, the characteristic index æ and the Bloch phase ϕ in the first and second approximations take the forms

$$æ_{1,2} = q\frac{\pi}{d} + i\gamma_{1,2}\,, \quad \phi_{1,2} = q\,\pi + i\gamma_{1,2}\,d\,, \tag{18}$$

where

$$\gamma_1(k) = \sqrt{|p_q(k)|^2 - \delta_q^2}\,, \quad \gamma_2(k) = \sqrt{|p_q(k)|^2 - \eta_q^2} \tag{19}$$

and

$$\eta_q = \delta_q + \frac{d}{2\pi}\sum_{m\neq q=-\infty}^{m=+\infty}\frac{|p_m|^2}{m-q-\delta_q\,d/\pi}\,. \tag{20}$$

In forbidden bands, where $|p_q| > |\delta_q|$ (first approximation), or $|p_q| > |\eta_q|$ (second approximation), $\gamma_{1,2}$ is a real positive number. In allowed bands, where $|p_q| < |\delta_q|$ (first approximation), or $|p_q| < |\eta_q|$ (second approximation), $\gamma_{1,2}$ is a pure imaginary number: $\gamma_1 = i|\gamma_1|$, if $\delta_q < 0$ and $\gamma_1 = -i|\gamma_1|$, if $\delta_q > 0$; $\gamma_2 = i|\gamma_2|$, if $\eta_q < 0$ and $\gamma_2 = -i|\gamma_2|$, if $\eta_q > 0$. These formulae determine the position and width of the forbidden band for the TM mode at a given angle of incidence θ_0. Similar formulae apply to the TE mode at the same angle but with different coefficients p_m as in [9].

At normal incidence the distinction between TM and TE modes disappears; to be more accurate the coupling coefficients for the two polarizations are related by $p_m^{TM} = -p_m^{TE}$. At increasingly oblique angles the forbidden band of the TE mode widens (if all other parameters of the slab are fixed), whereas the forbidden band of the TM mode narrows. The center of the forbidden band shifts to higher wavenumber k (to higher frequencies). Therefore, an omnidirectional forbidden band for both TE and TM polarizations occurs if there is an overlap between the forbidden band at normal incidence and the forbidden band of the TM mode at 90°. As a result, in a first approximation the right k_r and left k_l boundaries of the q-th omnidirectional forbidden band can be found from the equations

$$\begin{aligned} k_r\,n_{av} - q\,\pi/d &= |p_q(k_{r,\,\theta_0=0})|\,, \\ q\,\pi/d - k_l\,n_{min,av} &= |p_q(k_{l,\,\theta_0=90^\circ})|\,, \end{aligned} \tag{21}$$

where

$$n_{av} = \frac{1}{d}\int_0^d n(z')\,dz'\,,$$

$$n_{min,av} \;=\; \frac{1}{d}\int_0^d \sqrt{n^2(z')-n_0^2}\,dz' \,. \qquad (22)$$

Therefore, the center k_c of the q-th omnidirectional reflection band and its relative bandwidth Δ_q are given by

$$\begin{aligned} k_c &= \frac{\pi\, q}{2d}\left(\frac{1}{n_{av}}+\frac{1}{n_{min,av}}\right) \\ &\quad + \frac{|p_q(k_{r,\,\theta_0=0})|}{2n_{av}} - \frac{|p_q(k_{l,\,\theta_0=90^\circ})|}{2n_{min,av}}\,, \\ \Delta_q k_c &= \frac{\pi\, q}{d}\left(\frac{1}{n_{av}}-\frac{1}{n_{min,av}}\right) \\ &\quad + \frac{|p_q(k_{r,\,\theta_0=0})|}{n_{av}} + \frac{|p_q(k_{l,\,\theta_0=90^\circ})|}{n_{min,av}}\,. \qquad (23)\end{aligned}$$

The reflection and transmission amplitudes for a wave incident on a matched periodic structure in the first and second approximations take the form

$$\begin{aligned} r_B^{(1)} &= \frac{-p_q^*\sinh(\gamma_1 L)}{\gamma_1\cosh(\gamma_1 L)-i\delta_q\sinh(\gamma_1 L)}\,, \\ t_B^{(1)} &= \frac{\gamma_1 e^{i\pi Nq}}{\gamma_1\cosh(\gamma_1 L)-i\delta_q\sinh(\gamma_1 L)}\,; \qquad (24)\end{aligned}$$

$$\begin{aligned} r_B^{(2)} &= \frac{\left(-p_q^*-2i\,\eta_q u^*+p_q u^{*2}\right)\sinh(\gamma_2 L)}{(1-|u|^2)\,\gamma_2\cosh(\gamma_2 L)-i\,[(1+|u|^2)\eta_q-2\,\mathrm{Im}(p_q u^*)]\sinh(\gamma_2 L)}\,, \\ t_B^{(2)} &= \frac{(1-|u|^2)\,\gamma_2\,e^{i\pi Nq}}{(1-|u|^2)\,\gamma_2\cosh(\gamma_2 L)-i\,[(1+|u|^2)\eta_q-2\,\mathrm{Im}(p_q u^*)]\sinh(\gamma_2 L)}\,, \qquad (25)\end{aligned}$$

where

$$u = -\frac{id}{2\pi}\sum_{m=-\infty,\,m\neq q}^{m=+\infty}\frac{p_m}{m-q-\delta_q\,d/\pi}\,. \qquad (26)$$

By matched, we mean that the refractive index is continuous across the exterior boundaries at $z=0$ and $z=L$, i.e. there is no Fresnel reflection from them. The reflection r_Σ and transmission t_Σ amplitudes for an arbitrary (non-matched) periodic structure can be found from the matrix equation

$$\begin{pmatrix}1\\ r_\Sigma\end{pmatrix} = \begin{pmatrix}1/t_0 & r_0/t_0\\ r_0/t_0 & 1/t_0\end{pmatrix}\begin{pmatrix}1/t_B & r_B^*/t_B^*\\ r_B/t_B & 1/t_B^*\end{pmatrix}\times \begin{pmatrix}1/t_f & r_f/t_f\\ r_f/t_f & 1/t_f\end{pmatrix}\begin{pmatrix}t_\Sigma\\ 0\end{pmatrix}\,, \qquad (27)$$

where the Fresnel reflection $r_{0,f}$ and transmission coefficients $t_{0,f}$ for TM waves are responsible for the wave transformation on the boundaries of the structure.

The expressions (24) for the reflection and transmission coefficients in the first approximation of the semiclassical coupled wave theory have the same form as those in the conventional coupled wave theory [7, 8, 11], if we take into account the difference between the positions of the Bragg resonances described above, and the magnitudes of coupling coefficients in the two theories. Due to these differences, as we shall see in the next section, the first approximation of our semiclassical theory already gives eminently reasonable results in cases where the conventional theory fails. The second approximation of the semiclassical theory gives good agreement (within 10%) with exact numerical results even in the most unfavourable situations.

III. BI-LAYER PHOTONIC CRYSTAL

To illustrate our semiclassical coupled wave theory, we consider a two-layered periodic medium with real refractive indices n_1 and n_2 and layer thicknesses d_1 and d_2 such that $d=d_1+d_2$, as shown in Figure 1.

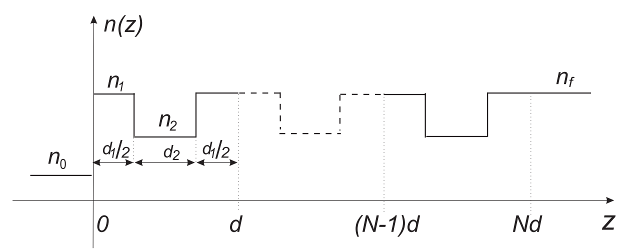

FIG. 1: Two-layered periodic dielectric structure (bi-layer photonic crystal).

According to (12) the effective averaged refractive index of this slab is

$$n_{e,av} = \frac{\sqrt{n_1^2-\beta^2}\,d_1+\sqrt{n_2^2-\beta^2}\,d_2}{d}\,. \qquad (28)$$

The first (integral) term in (14) is zero. The contributions to the second term come from the points $z_1=d_1/2$ and $z_2=d_1/2+d_2$. Summing them up, we obtain

$$p_m = \frac{i}{d}\ln\left[\frac{\sqrt{n_2^2-\beta^2}\,n_1^2}{\sqrt{n_1^2-\beta^2}\,n_2^2}\right]e^{-im\pi} \times\sin\left[\frac{d_2}{d}\left[m\pi+kd_1\left(\sqrt{n_2^2-\beta^2}-\sqrt{n_1^2-\beta^2}\right)\right]\right]. \quad (29)$$

As a result, the relative bandwidth of the q-th omnidirectional reflection band according to eqs. (23) is

$$\Delta_q \approx 2\,\frac{q\pi\,(1-a)\,/d + |p_q(k_{q,\,\theta_0=0})| + a\,|p_q(k_{q,\,\theta_0=90^\circ})|}{q\pi\,(1+a)\,/d + |p_q(k_{q,\,\theta_0=0})| - a\,|p_q(k_{q,\,\theta_0=90^\circ})|},\quad a \equiv \frac{n_{10}\,d_{12} + n_{20}}{\sqrt{n_{10}^2-1}\,d_{12} + \sqrt{n_{20}^2-1}}\,, \tag{30}$$

where $n_{10} = n_1/n_0$, $n_{20} = n_2/n_0$, $d_{12} = d_1/d_2$, and we take into account that

$$\begin{aligned} |p_q(k_{l,\,\theta_0=90^\circ})| \approx |p_q(k_{q,\,\theta_0=90^\circ})| &= \frac{1}{d}\left|\ln\left(\frac{\sqrt{n_{20}^2-1}\,n_{10}^2}{\sqrt{n_{10}^2-1}\,n_{20}^2}\right)\sin\left(\frac{q\pi}{1+\sqrt{n_{10}^2-1}\,d_{12}/\sqrt{n_{20}^2-1}}\right)\right|,\\ |p_q(k_{r,\,\theta_0=0})| \approx |p_q(k_{q,\,\theta_0=0})| &= \frac{1}{d}\left|\ln\left(\frac{n_{10}}{n_{20}}\right)\sin\left(\frac{q\pi}{1+n_{10}d_{12}/n_{20}}\right)\right|. \end{aligned} \tag{31}$$

One sees that the relative bandwidth of the q-th omnidirectional band (when it exists) depends on just three ratios: n_{10}, n_{20} and d_{12}. For each choice of materials, i.e. for each choice of n_{10} and n_{20}, there is a value of d_{12} that maximizes the relative bandwidth. Moreover, for a given ambient medium n_0 we can obtain the widest possible relative bandwidth of the q-th omnidirectional band if we maximize the above function with respect to all three parameters simultaneously.

Let us consider a specific example. For the tin sulfide/silica ($n_1 = 2.6$, $n_2 = 1.46$) structure in air ($n_0 = 1.0$) on a substrate with $n_f = 2.6$, the first omnidirectional reflection band is centered at the frequency $\nu_c = 4.49\times10^{14}$ Hz ($\lambda_c = 668$ nm) with the relative bandwidth $\Delta_1 = 9.2\%$ if the thicknesses of the layers are $d_1 = 80$ nm and $d_2 = 115$ nm. These parameters correspond to those in an experiment of Fink *et al* [14]. Keeping the first material the same ($n_1 = 2.6$), we obtain from eq. (30) that the optimal second material should have $n_2 = 1.5$ and the ratio of the layer thicknesses should be $d_{12} = 8/11$, rather than $d_{12} = 16/23$. This ratio provides relative bandwidth $\Delta_1 = 9.4\%$. To obtain omnidirectional reflection centered at the same frequency $\nu_c = 4.49\times10^{14}$ Hz, we need $d_1 = 80$ nm and $d_2 = 110$ nm. In Fig. 2 we show the Bloch phase ϕ and reflection coefficient for a structure of $N = 6$ periods. The exact results were obtained numerically by the transfer matrix method. We can see that even the first approximation of the semiclassical coupled wave theory works well for all frequencies of the incoming waves.

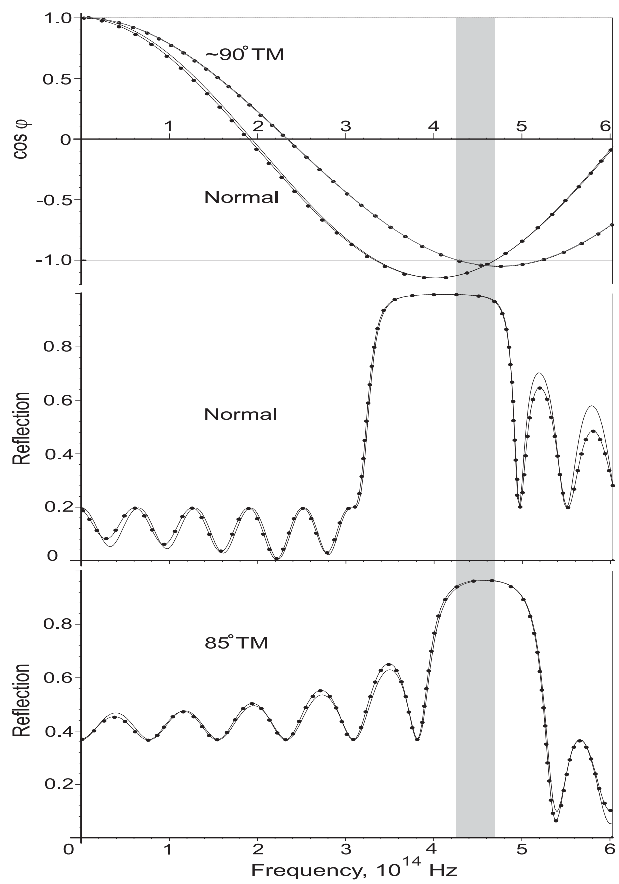

FIG. 2: Bloch phase and reflection vs. frequency for TM mode at normal and 85° angle of incidence on the two-layered periodic structure with the parameters described in text; exact numerical results (solid line); first approximation of the semiclassical theory (thin line with points). The shaded region shows the omnidirectional bandgap.

IV. HARMONIC PERIODIC STRUCTURE

As a second application of our theory, we consider a periodic structure with a harmonic refractive index profile of the form $n(z) = n_{av} + n_A \sin(2\pi z/d)$, see Fig. 3. Initially we take $n_{av} = 1.965$ and $d = 190$ nm as in the previous example of a bilayer photonic crystal. The number of periods $N = 6$ and the refractive indices of an ambient media $n_0 = 1$ and a substrate $n_f = 2.6$ correspond to that example as well. For a moderate refractive index modulation $n_A = 0.5$ the results are shown in Fig. 4. Again, the first approximation of the semiclassical coupled wave theory is in good agreement with the exact numerical solutions.

For the second harmonic structure we take $n_{av} = 3$ and $d = 124.5$ nm (we keep the same product $n_{av}\,d$ as in the previous example) and consider the very deep modulation $n_A = 1.5$. The results are shown in Fig. 5. The

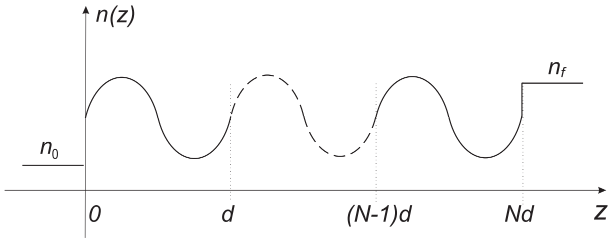

FIG. 3: Harmonic periodic dielectric structure

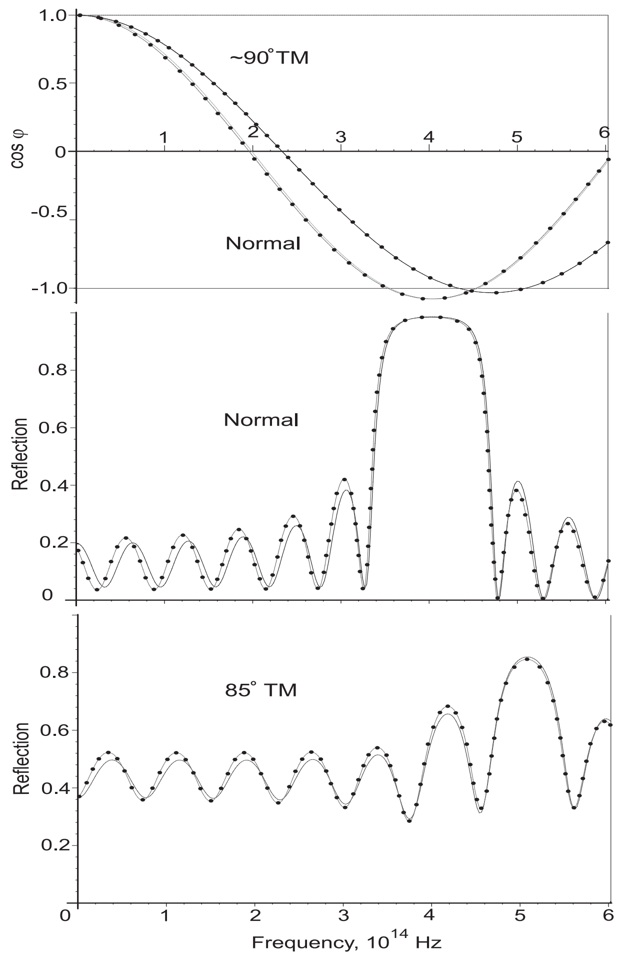

FIG. 4: Bloch phase and reflection vs. frequency for TM mode at normal and 85° angle of incidence on the harmonic periodic structure with a moderate ($n_A = 0.5$) refractive index modulation; the parameters are as described in text; exact numerical results (solid line); first approximation of the semiclassical theory (thin line with points).

first approximation, especially for normal incidence, is satisfactory only in forbidden bands. However, the second approximation works well for all frequencies of the incoming waves. Also, this example shows that for TM waves, the first approximation of our semiclassical coupled wave theory works better for large incident angles

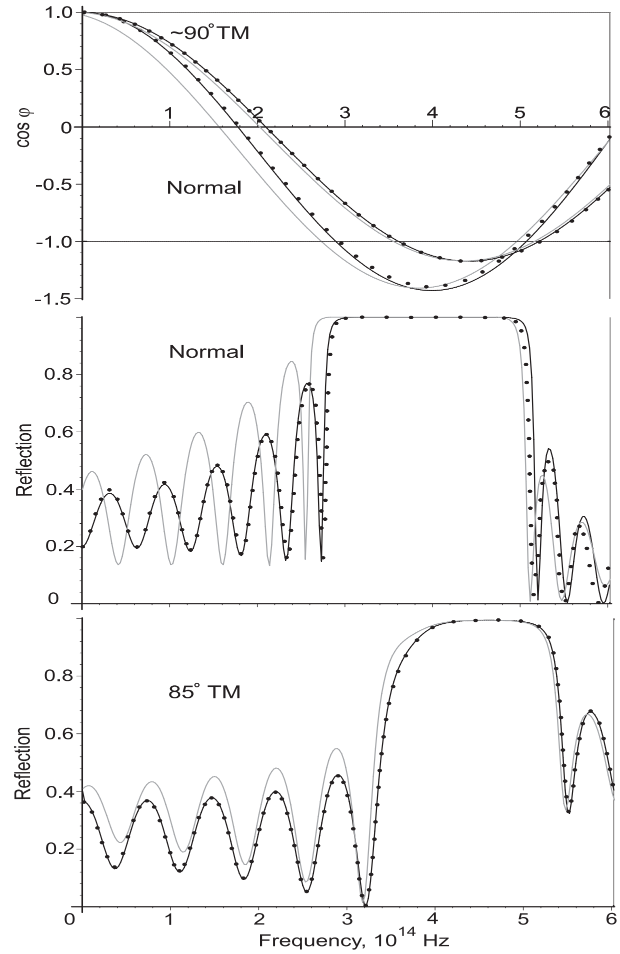

FIG. 5: Bloch phase and reflection vs. frequency for TM mode at normal and 85° angle of incidence on the harmonic periodic structure with a deep ($n_A = 1.5$) refractive index modulation; the parameters are as described in text; exact numerical results (solid line); first approximation of the semiclassical theory (gray solid line); second approximation of the semiclassical theory (points).

θ_0 than for small ones. This is contrary to the case of TE waves; see [9].

V. CONCLUSIONS

The semiclassical coupled wave theory [9] has been extended to TM waves. The results obtained show that simple approximate anlytical solutions (the first approximation of our theory) provide all significant characteristics of the exact solutions in many cases. However, if the modulation of the refractive index of the periodic structure is very deep, one needs to go to the second approximation, which is based on the Bogolyubov averaging method. In the future, we plan to extend the semiclassical coupled wave theory to absorptive media.

Acknowledgements

We are grateful to NSERC-Canada for continued support through Discovery Grant SAPIN-3198 (DWLS, GVM) and to DGES-Spain through grant BFM2001-3710 (JM). This work was carried out as part of CERION-2, EU thematic network IST-2001-39059.

---

[1] Y. Fink, J. N. Winn, S. Fan, C. Chen, J. Michel, J. D. Joannopoulos, and E. L. Thomas, "A dielectric omnidirectional reflector", Science **282**, 1679 (1998).

[2] D. N. Chigrin, A.V. Lavrinenko, D. A. Yarotsky, and S.V. Gaponenko, "Observation of total omnidirectional reflection from a one-dimensional dielectric lattice", Appl. Phys. A **68**, 25 (1999).

[3] J. Lekner, "Omnidirectional reflection by multilayer dielectric mirrors", J. Opt. A: Pure Appl. Opt. **2**, 349 (2000).

[4] D. Bria, B. Djafari-Rouhani, E. H. El Boudouti, A. Mir, A. Akjouj, and A. Nougaoui, "Omnidirectional optical mirror in a cladded-superlattice structure", J. Appl. Phys. **91**, 2569 (2002).

[5] Hyun-Yong Lee and Takafumi Yao, "Design and evaluation of omnidirectional one-dimensional photonic crystals", J. Appl. Phys. **93**, 819 (2003).

[6] H. Kogelnik, "Coupled wave theory for thick hologram gratings", Bell Syst. Tech. J. **48**, 2909 (1969).

[7] Amnon Yariv, Pochi Yeh, *Optical Waves in Crystals* (John Wiley & Sons, New-York, 1984), Chap. 6.

[8] Pochi Yeh, *Optical Waves in Layered Media* (John Wiley & Sons, New-York, 1988), Chap. 8.

[9] G. V. Morozov, D. W. L. Sprung, and J. Martorell, "Semiclassical coupled wave theory and its application to TE-waves in 1-D photonic crystals", Phys. Rev. E **69**, in press (2004).

[10] N. N. Martynov and S. N. Stolyarov, "Theory of wave propagation in periodic structures", Kvant. Elektron. **5**, 1853 (1979) [Sov. J. Quantum Electron. **8** 1056 (1979)].

[11] S. Yu. Karpov and S. N. Stolyarov, "Propagation and transformation of electromagnetic waves in one-dimensional periodic structures", Usp. Fiz. Nauk **163**, 63 (1993) [Phys. Usp. **36** 1 (1993)].

[12] N. N. Bogoliubov and Y. A. Mitropolsky, *Asymptotic Methods in the Theory of Non-Linear Oscillations* (Gordon and Breach, New-York, 1961), Chap. 5.

[13] V. L. Ginzburg, *Propagation of Electromagnetic Waves in Plasma* (Gordon and Breach, New-York, 1961), Chap. 4.

[14] M. Deopura, C.K. Ullal, B. Temlkuran, and Y. Fink, "Dielectric omnidirectional visible reflector", Optics Letters **26**, 1197 (2001).